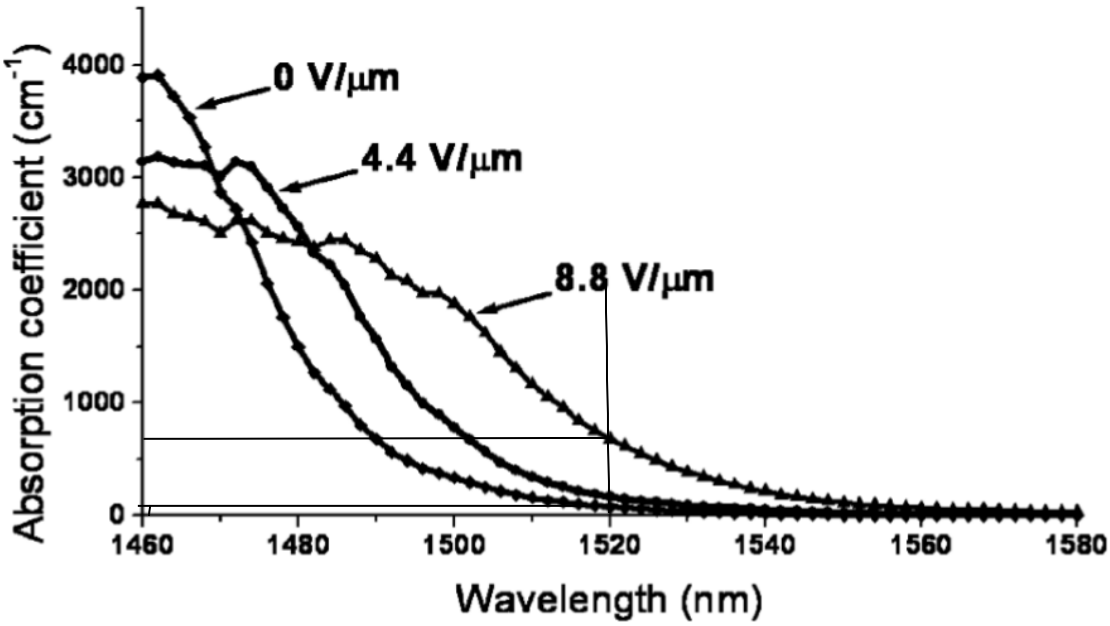
SOLUTIONS

Exercise 1

We want to operate an electro-absorption modulator AM operating at $\lambda = 1520\text{ nm}$. the device is based on InGasAsP/InP and the absorption as been experimentally characterized. The results are given in the following graph which shows the absorption as a function of wavelength for different values of electric field in the active region (from Helman et al. JSTQE 2005). The active region is 350 nm wide and 100 μm long and you can assume that when you apply a voltage across the width of the device, and that the entire voltage drops in this region only.



(a) What are the voltages required to obtain the three curves shown in the graph ?

Assume a uniform field distribution and given that the voltage is applied across a $w = 350\text{ nm}$ region than we have that $E = V/w$. T

The voltages are therefore 1.54 V and 3.08 V for the fields of 4.4 $\text{V}/\mu\text{m}$ and 8.8 $\text{V}/\mu\text{m}$, respectively

(b) Calculate the transmission of the modulator for 0 V and for the maximum voltage found in part (a).

From the graph we can read the absorption for 0V and for 3.08 V. They are approximately 100 cm^{-1} and 700 cm^{-1}. As such the transmissions are:

$T(0) = \exp(-100 \times 0.01) = 0.37$

$T(3.08\text{ V}) = \exp(-700 \times 0.01) = 9.1 \cdot 10^{-4}$

(c) What is the insertion loss of the device in dB (light loss even when the device is at 0 V) ?

When there is no voltage the transmission is of 0.37, which means a 4.3 dB insertion loss.

(d) What is the extinction ratio of the device ?

It is given by $R_{on/off} \approx 4.34L\left[\alpha\left(V_{app}\right) - \alpha(0)\right] = 26 \text{ dB}$

Exercise 2

A lithium niobate ($LiNbO_3$) Mach-Zehnder modulator has an electrode length of L = 1.5 cm and an electrode gap of $d = 10$ µm.

For the specific crystal orientation and polarization of our device, we have:

- Operation wavelength is 1550 nm
- Refractive index of $LiNbO_3$ of 2.2
- Pockels coefficient 30 pm/V

(a) Calculate V_π for the given parameters

By definition $V_\pi = \frac{d}{L}\frac{\lambda_0}{rn^3}$. Using the given values we get that

$$\boldsymbol{V_\pi \approx 3.24 \text{ V}}$$

(b) If the modulator is driven by a voltage source of 5 V peak-to-peak , what is the resulting phase shift of the MZI ?

The phase shift is given by : $\Delta\phi = \pi\frac{V}{V_\pi}$.

We get that $\boldsymbol{\Delta\phi \approx 4.85 \text{ rad}}$

We upgrade the modulator to a **push-pull operation: it** operates by applying equal and opposite voltage changes to the two arms of the interferometer, say V and $-V$.

(c) Derive the equation describing the relation between the output field E_{out2} and the input field E_{in}

Doing the derivation by following the same procedure as seen in class we get that :

$$E_{out2} = \frac{i}{2}E_{in}\exp\left[\frac{i}{2}(\phi_1 + \phi_2)\right]\left[\exp\left[\frac{i}{2}\left(\phi_1 - \phi_2 - \pi\frac{2V}{V_\pi}\right)\right] + \exp\left[-\frac{i}{2}\left(\phi_1 - \phi_2 - \pi\frac{2V}{V_\pi}\right)\right]\right]$$

$$E_{out2} = iE_{in}\exp\left[\frac{i}{2}(\phi_1 + \phi_2)\right]\cos\left[\frac{1}{2}\left(\phi_1 - \phi_2 - \pi\frac{2V}{V_\pi}\right)\right]$$

(d) Show that the output power is given by: $P_{out} = P_{in}\cos^2\left(\frac{\Delta\phi}{2}\right)$ with $\Delta\phi = \phi_1 - \phi_2 - \pi\frac{2V}{V_\pi}$

We have that $P_{out} = |E_{out}|^2 = |E_{in}|^2\cos^2\left(\frac{1}{2}\left(\phi_1 - \phi_2 - \pi\frac{2V}{V_\pi}\right)\right)$

Hence $P_{out} = P_{in}\cos^2\left(\frac{\Delta\phi}{2}\right)$

(e) What can you say about the chirp $-d\phi_{out}/dt$? Compare to the single arm modulator.

The phase is given by $\phi_{out} = \frac{1}{2}(\phi_1 + \phi_2)$. We see that there is no change in phase with voltage, Hence there is not chirp while the single arm modulator had a chirp.

Exercise 3

We have seen in class that modulators can be integrated on a chip. One of the elements required is a power coupler where light can go from one waveguide to another. A coupler can be made from simply putting two waveguides side by side: if two waveguides are sufficiently close such that their fields overlap, light can couple from one into the other.

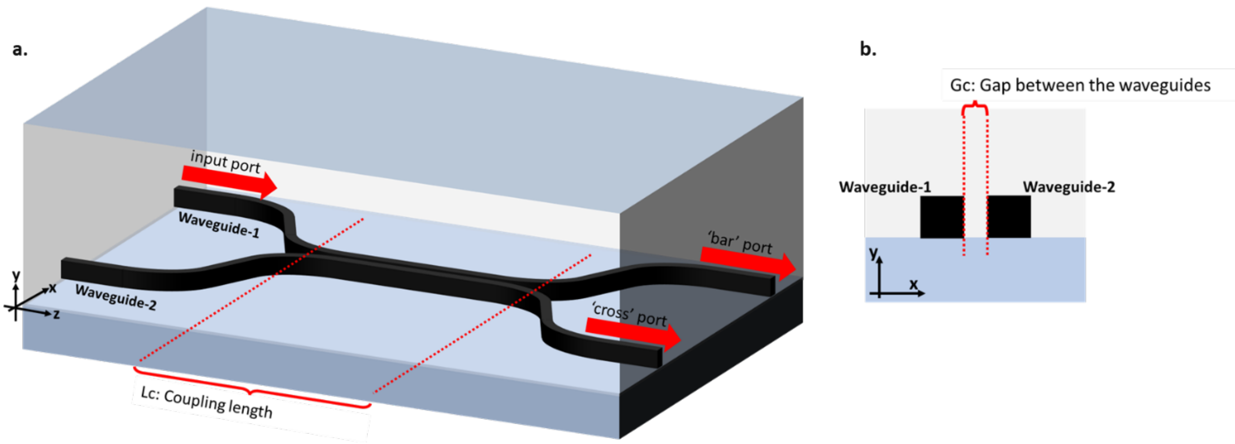


Figure 1 a. Schematic for the integrated coupler where BOX-oxide(light blue), cladding(grey) and waveguides(black). b. Cross-sectional view of the waveguide in the coupling region

By using coupled mode theory, transmitted power at each port can be determined:

$$P_{bar}(z) = \left((\cos(gz))^2 + \frac{\Delta\beta}{2}\frac{(\sin(gz))^2}{g^2}\right)P_{in}$$

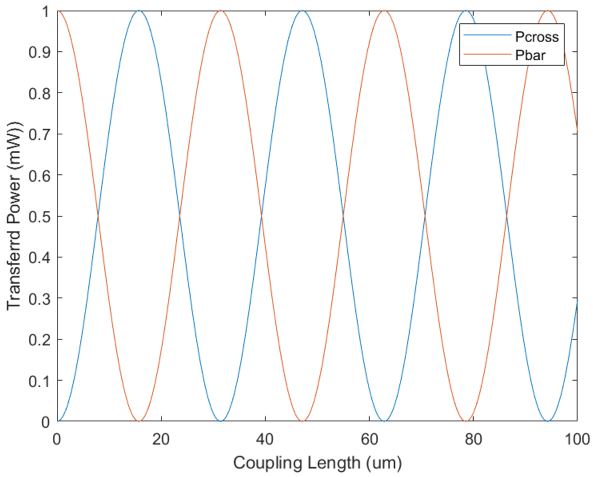
$$P_{cross}(z) = \kappa^2\frac{(\sin(gz))^2}{g^2}P_{in}$$

$$g^2 = \kappa^2 + \left(\frac{\Delta\beta}{2}\right)^2$$

Where κ is the coupling coefficient, $\Delta\beta = \beta_2 - \beta_1$ is the phase mismatch per unit length, z is the propagation length within the coupling region.

κ is large when the gap between the waveguides (Gc) is small and also depends on how confined the light is within the waveguide. $\Delta\beta$ is determined by dispersion properties of each waveguide. When two waveguides are identical, the coupler is said to be phase-matched as $\Delta\beta$ is zero.

(a) For κ = 0.1 µm^{-1}, using matlab plot the power versus coupling length in the 'bar' port and in the 'cross' port for the incidence power of 1 mW.



(b) For this configuration, pinpoint the minimum coupling length (L_c) to obtain a 3-dB coupler?

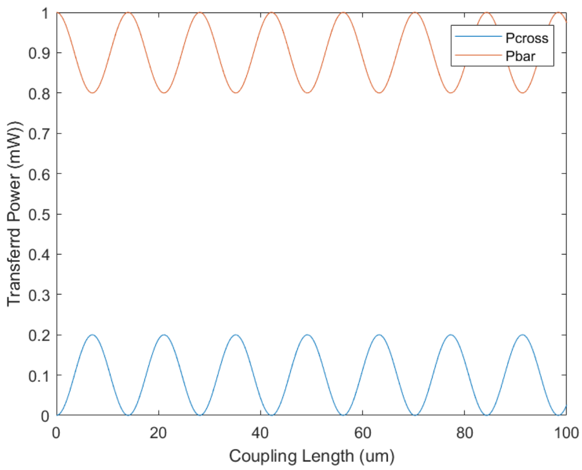
We can see that half the power is in the cross and half in the bar for a length of 7.75um

(c) For this configuration, pinpoint the minimum coupling length (L_c) to obtain a complete optical switch, i.e. the transmission from input port to the 'cross' port is 100%?

We can see that all the power is in the cross and none in the bar for a length of 15um

Many optical components require specific dispersion properties, and the waveguide dimensions may not be altered easily. In the next subsection, we investigate the response when the waveguides within the coupler have different dimensions. When β_2 and β_1 are not identical, this is called phase mismatched configuration.

(d) Repeat part (d) where β_1 is 8.1 μm^{-1} and β_2 is 8.5 μm^{-1}.

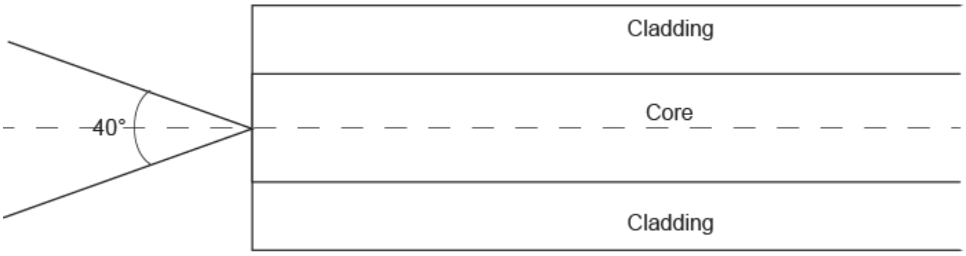


(e) Comment on whether it is possible to use this phase-mismatched coupler as a complete switch.

Obviously we cannot !

Exercise 4

You need a step-index multi-mode fiber that can guide all incoming light (via total internal reflection) within a cone of 40 degrees and transmit a 10 Mb/s signal 500 m. The refractive index of the core is 1.5 and the medium outside the fiber is air.



(a) Choose a value for the refractive index of the cladding to satisfy the coupling condition.
We need TIR therefore we apply Snell's law at the input face:

$$sin(\alpha) = n_{core} sin(\beta)$$

$$\beta = 13.18°$$

We apply Snell's law at the core cladding interface:

$$n_{core} sin(\theta) = n_{cladding} sin(90°)$$

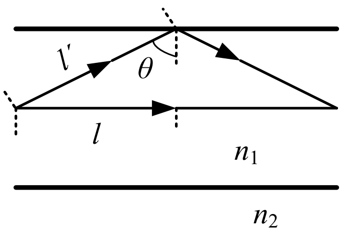
$$n_{core} sin(90° - \beta) = n_{cladding}$$

$$\boldsymbol{n_{cladding} = 1.4605}$$

(b) Let's estimate the pulse broadening ΔT due to the multimode propagation in a fiber of L, a core index n_1, a cladding index n_2 and critical angle θ_c. Base on the incident angles leading to the shortest and longest path, show that the ΔT (if assumed equivalent to the difference in time between the two path and that the velocity of propagation is c_0/n_1) is given by:

$$\Delta T = \frac{L}{c_0} \frac{n_1^2}{n_2} \Delta$$

The shortest traveling path for a fixed longitudinal length corresponds to the beam with normal incident angle to the fiber surface. On the other hand, the longest path is given by the beam with minimum incident angle denoted by θ in the figure, which is limited to the critical angle θ_c.



For the two beams traveling different paths l and l', the path length difference leads to a time delay equal to:

$$\Delta T' = \frac{n_1}{c_0}\left(\frac{l}{\sin\theta} - l\right)$$

where the term $\frac{c_0}{n_1}$ denotes light propagation speed in a medium with refractive index n_1 and the term in parenthesis indicates the difference in path length of two depicted beams. We used the fact that $\sin\theta = l/l'$

Therefore for a fiber with length L, the time delay ΔT between the shortest and the longest path is obtained by:

$$\Delta T = \frac{n_1}{c_0} L\left(\frac{1}{\sin\theta} - 1\right)$$

Using $\sin\theta = n_2/n_1$, we finally get that :

$$\boldsymbol{\Delta T = \frac{L}{c_0}\frac{n_1^2}{n_2}\Delta}$$

(c) Can this fiber fulfill the transmission criteria you require, assuming that such intermodal broadening dominates?

We check that this fiber can support 10 Mbit/s over 500m. We know that :

$$B\Delta T \leq 1 \quad \Longrightarrow \quad BL \leq \frac{c_0 n_2}{n_1^2 \Delta}$$

We have $\Delta = \frac{(n_1 - n_2)}{n_1} = 0.026$

There fore $L \leq \frac{c_0 n_2}{n_1^2 \Delta B} = 739.24 \text{ m}$

This fiber can work since at 10 Mb/s the maximum length would be longer than 500m

Exercise 5

A 2.5 Gb/s data link is set up on an optical fiber with 0.25 dB/ km attenuation. The light source used has a spectral width of 6 nm.

(a) If the mean optical power at the input is 1 mW, what is the mean optical power of the light signal after a 28 km transmission distance?

We do a power budget for the link, only considering fiber loss, to find the power P_{rec} after $L = 28$ km.

$$P_{rec} = P_{tr} - \alpha L$$

$$P_{rec} = 0 - (0.25 \times 28)$$

$P_{rec} = -7$ dBm or $P_{rec} = 0.1995$ mW

(b) If 25 km is the maximum reliable transmission distance, estimate the dispersion parameter of the optical fiber used in the system.

We can estimate the dispersion form the standard relation $BL|D|\Delta\lambda \leq 1$ and the known values: B = 2.5 Gb/s, $\Delta\lambda = 6\text{ nm}$ 6 nm and $L_{max} = 25\text{ km}$. At the maximum reliable distance the equation is equal to 1. We get:

$$|D| = \frac{1}{BL_{max}\Delta\lambda}$$

$$|D| = \frac{1}{(25 \times 10^9\text{s}^{-1})(25\text{ km})(6\text{ nm})}$$

$$|D| = 2.667\text{ ps/(nm} - \text{km)}$$

(c) Near the zero-dispersion wavelength λ_0, the dispersion parameter of the fiber can be expressed as a function of the dispersion slope S_0: $D(\lambda) = S_0(\lambda - \lambda_0)$. Given that $\lambda_0 = 1550\text{ nm}$ and $S_0 = 0.1\text{ ps/(nm}^2 \cdot \text{km)}$, estimate the center wavelength of the light source.

We are given λ_0= 1550 nm and S_0 = 0.1 ps/(nm^2.km) and have gotten a value of $|D|$. We can write:

$$|\lambda - \lambda_0| = \frac{|D|}{S_0}$$

$$\lambda = \lambda_0 \pm \frac{D}{S_0}$$

$$\lambda = 1550 \pm \frac{2.667}{0.1}$$

The operating wavelength is therefore 1576.7 nm or 1523.3 nm